Great Ideas in (theoretical) Computer Science

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Shiv Nadar Institution of Eminence

Nov 25, 2023

1. Coin tossing with social distance
2. How powerful are Computers
3. A measure of difficulty
4. Doing things randomly
5. Secret Communication
6. Exploiting Parallelism
7. TCS research in India

When does it qualify as Great

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Long-lasting Ubiquity Simplicity Beauty

Fairness using coin tossing

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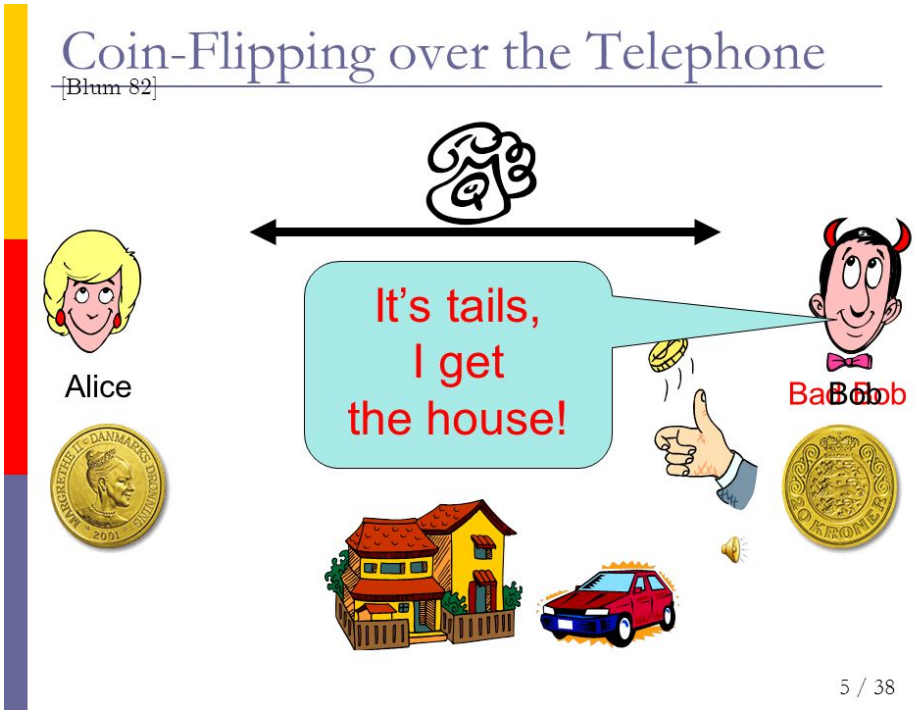
Coin tossing over telephone



Alice tosses, Bob calls

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Who goes first ?

- If Alice tells Bob the outcome, Bob can cheat.

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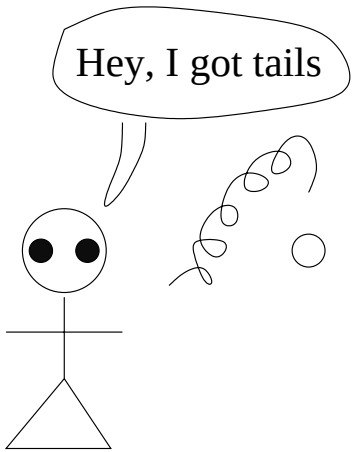
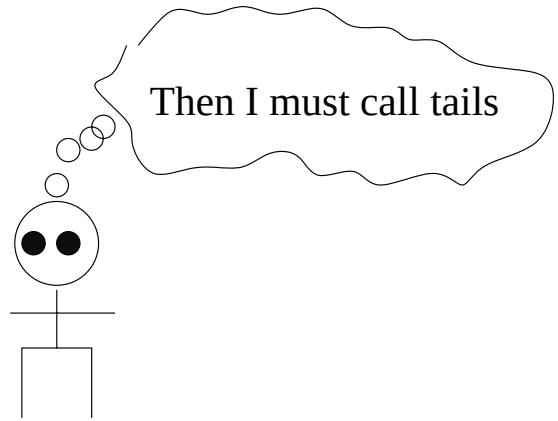


ALICE



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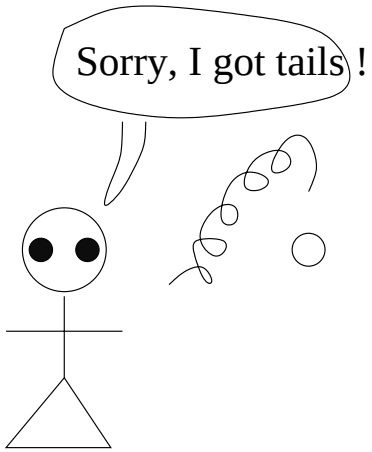

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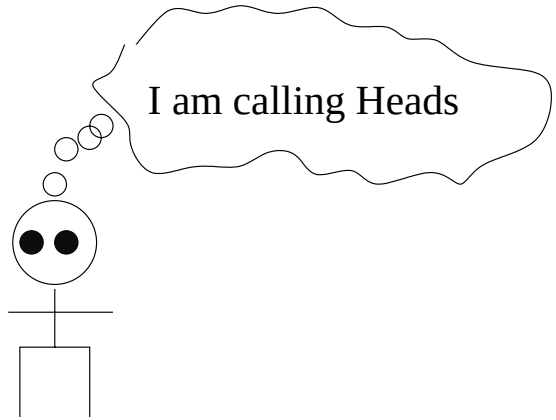

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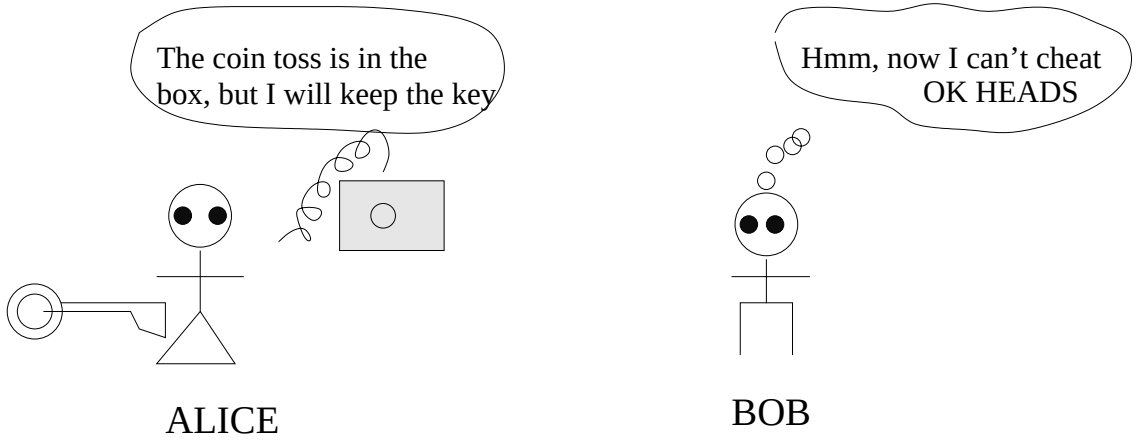

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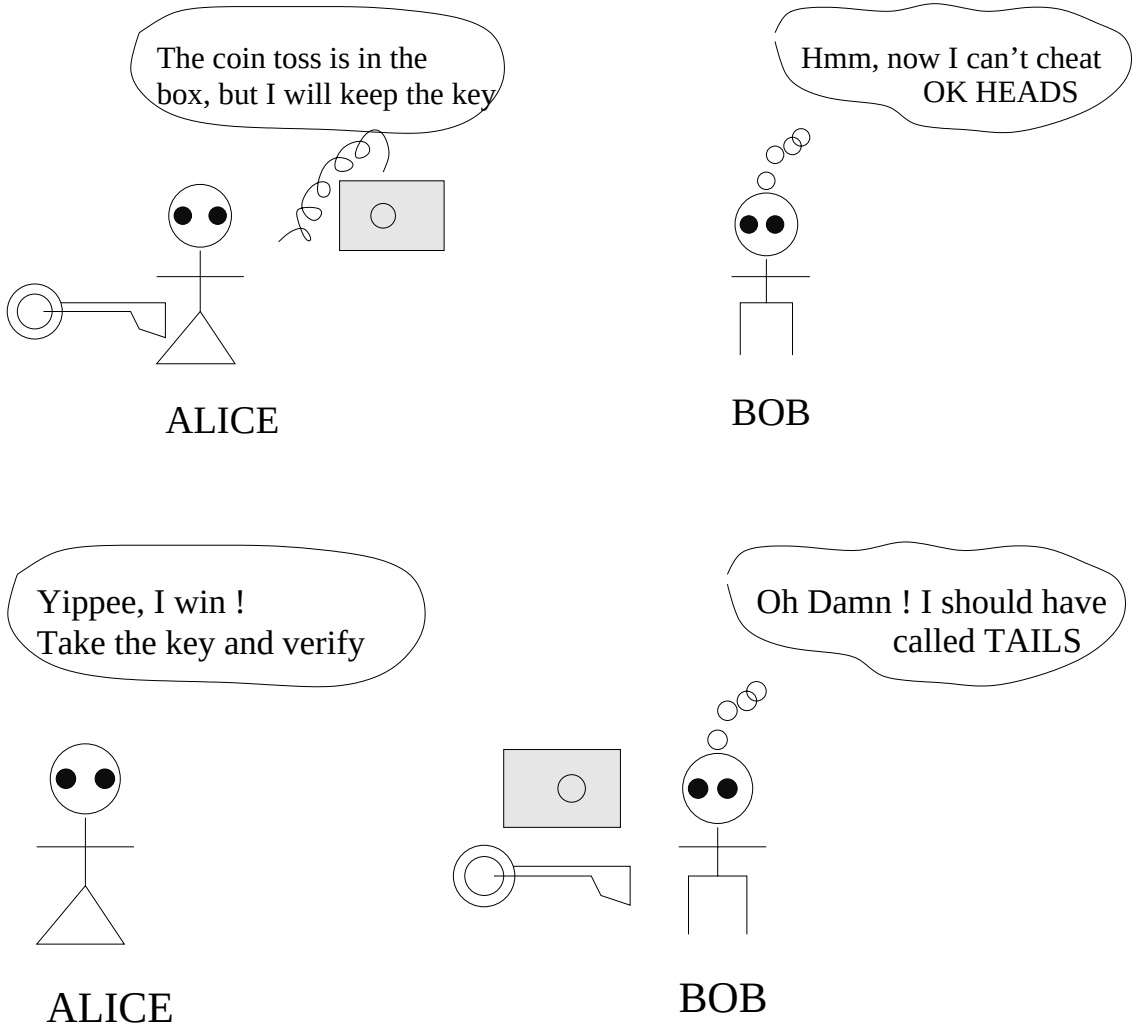
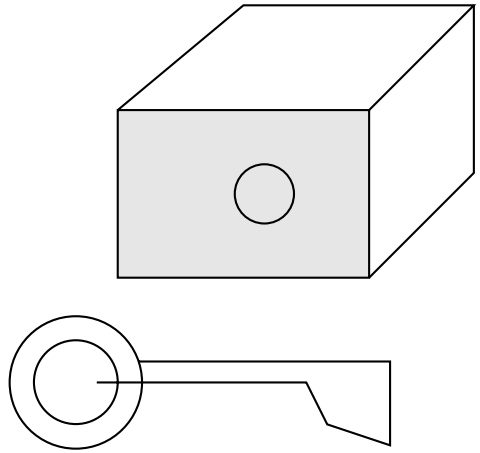


Virtual lock and key



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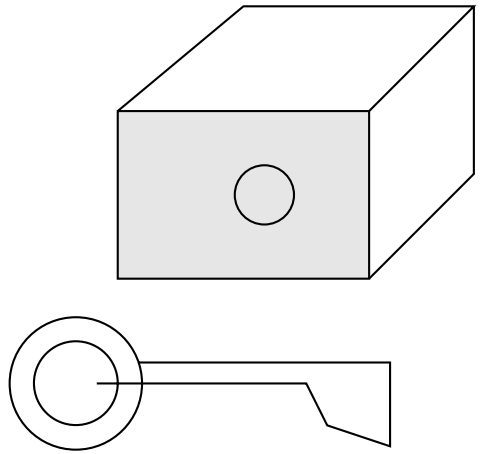

Theory of One-way functions to realise such a functionality.

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Would we have invested so much in building computers if we didn't feel convinced about its potential and universality !!

Some older machines

Leibnitz calculator 17th century

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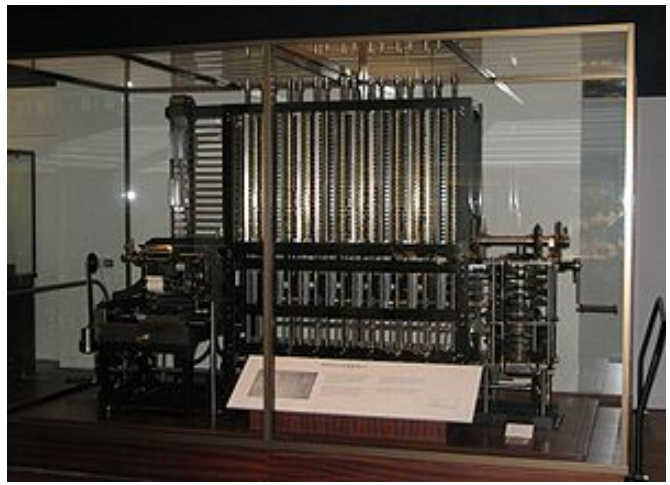
Babbage 19th century



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What can Computers do

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What can Computers do (or not do) from an intellectual standpoint ??

Speed is not pertinent for this question

Can Computers be used to solve anything ?

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1	0	100
2	01	00
3	110	11

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3 , , ,

A sequence : 110

B sequence : 11

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Post Correspondence Problem [PCP]

Is there a sequence (of any finite length) over $\{1, 2, 3\}$ such that if we concatenate the two sets of strings A, B, they become identical ?

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Does this contradict Wiles/Fermat result ? $x^i + y^i = z^i \quad i > 2$

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Goedel 1927

Logic is Incomplete - not powerful enough to discover all Theorems

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According to the thesis man and machine are not that different !

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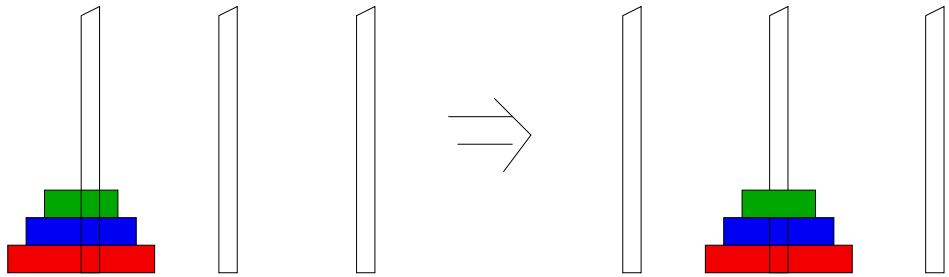
Measure of difficulty

if the best algororithm to solve a problem takes exponential steps, then it is unrealistic for solving large sized problems - a hopeless situation.

Tower of Hanoi



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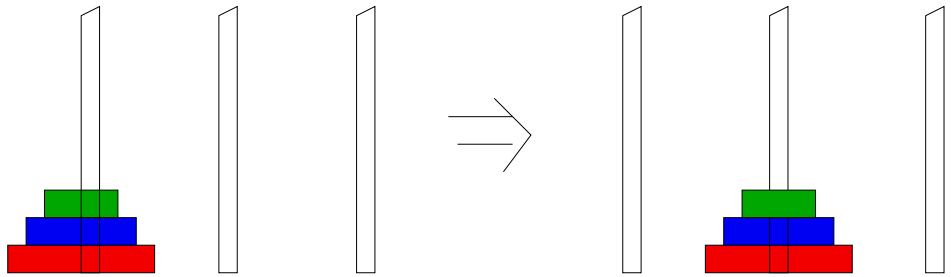

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Time to solve 64 disks

If the priests were able to move disks at a rate of one per second, using the smallest number of moves it would take them $2^{64} - 1$ seconds or roughly 585 billion years to finish, which is about 42 times the current age of the Universe.

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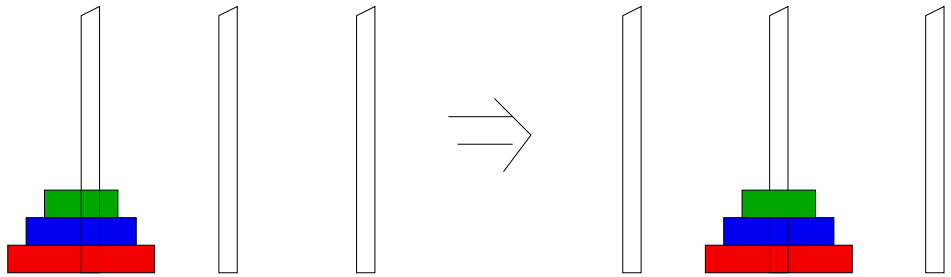

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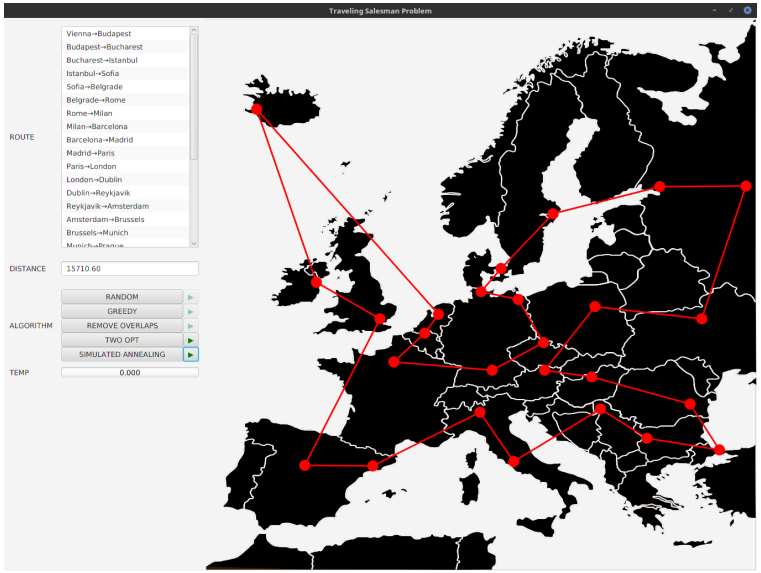
World will end by the time the game ends according to Brahma !

Travelling Salesman Problem



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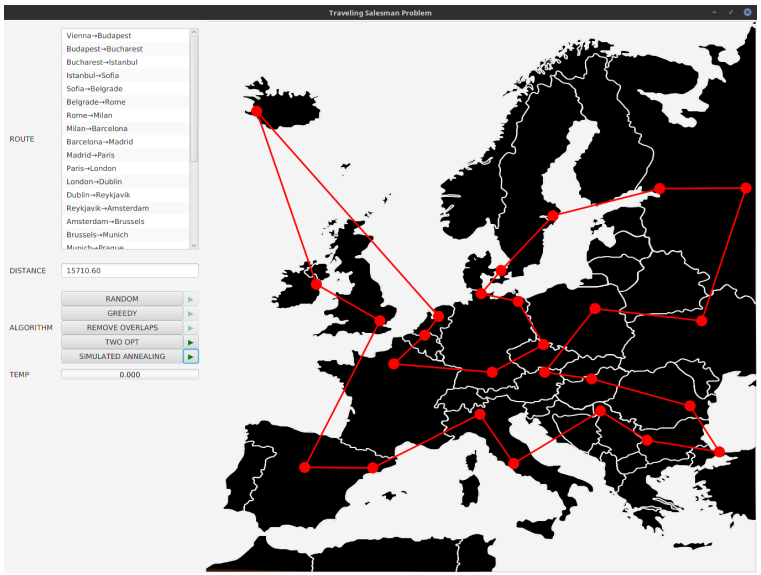

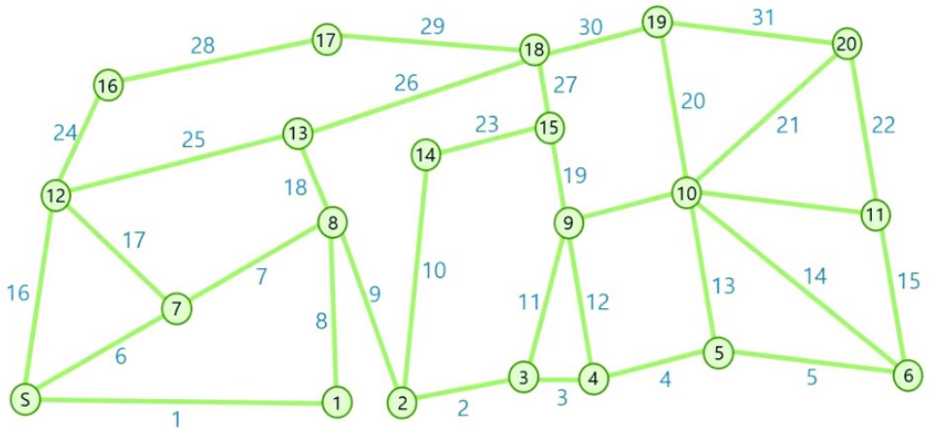

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Approximation algorithms

If a problem is hard but important to solve, we try to devise faster strategies/heuristics which may not be exact, but provides guarantees on its proximity to the actual best solution.

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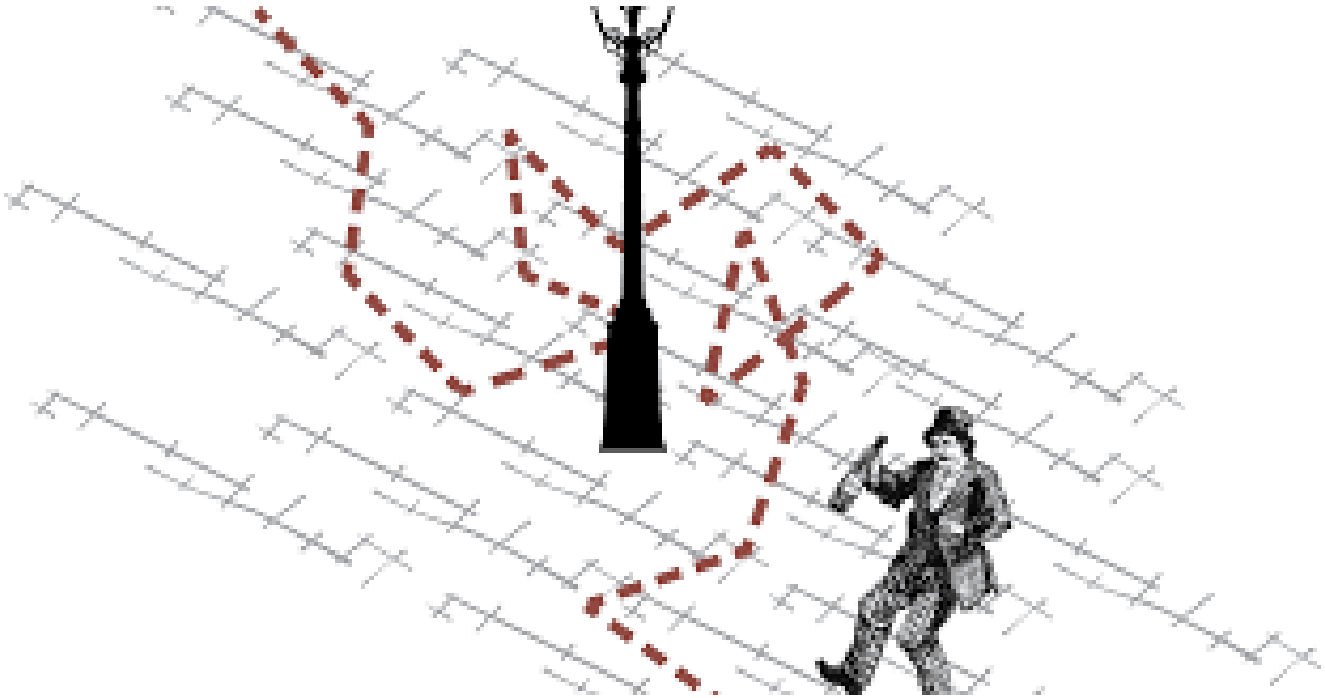
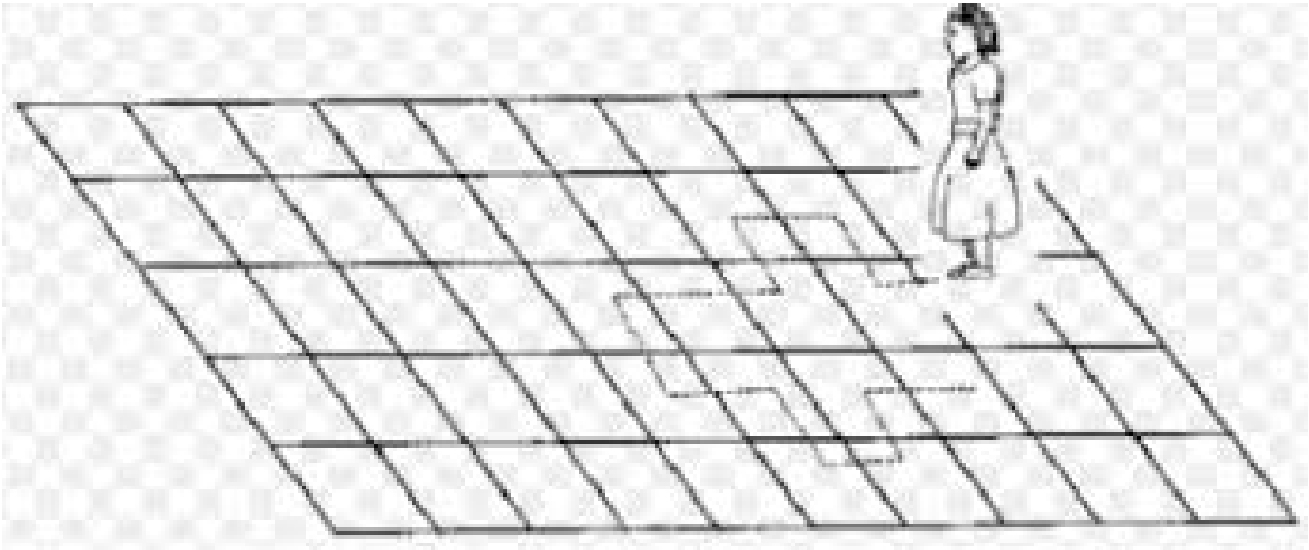
Theory is needed but lagging

When you can't control the outcome

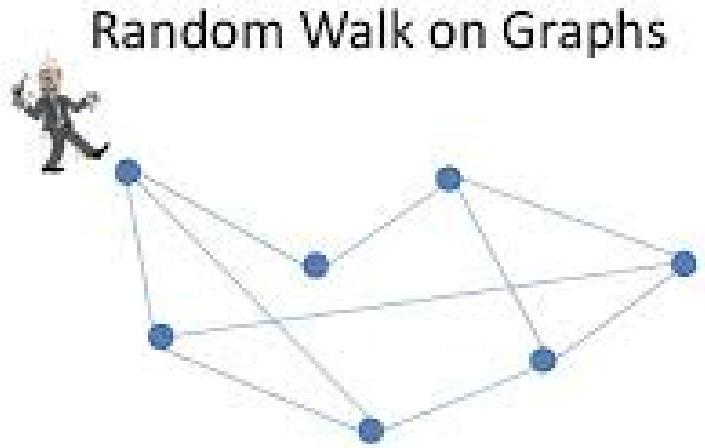


When there are few choices



Rat in a maze

Random walk works !



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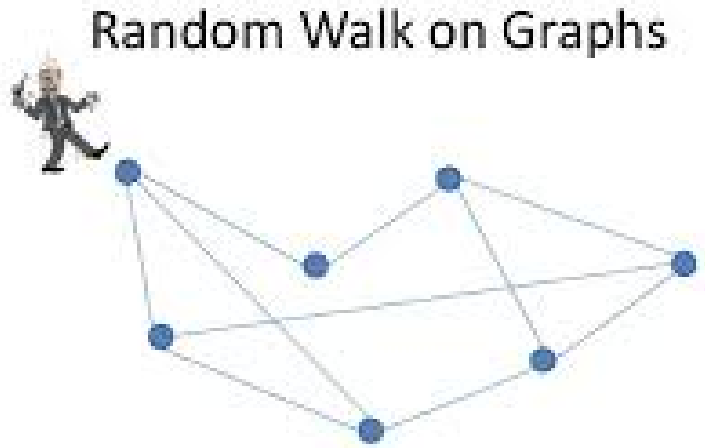

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Exploring without Maps

By choosing one of the outgoing edges at random from the current vertex, the traveller can visit any destination within $2V^3$ steps (expected).

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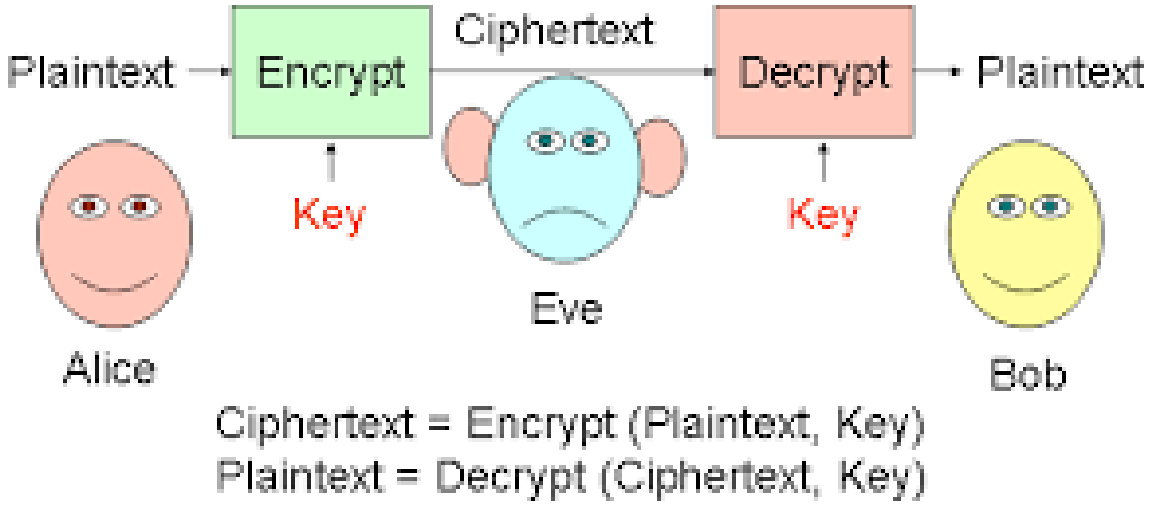
How do you share the code-book or the cipher design without sending this as another message !!

Setting up a cipher in the presence of Eve(sdropper)



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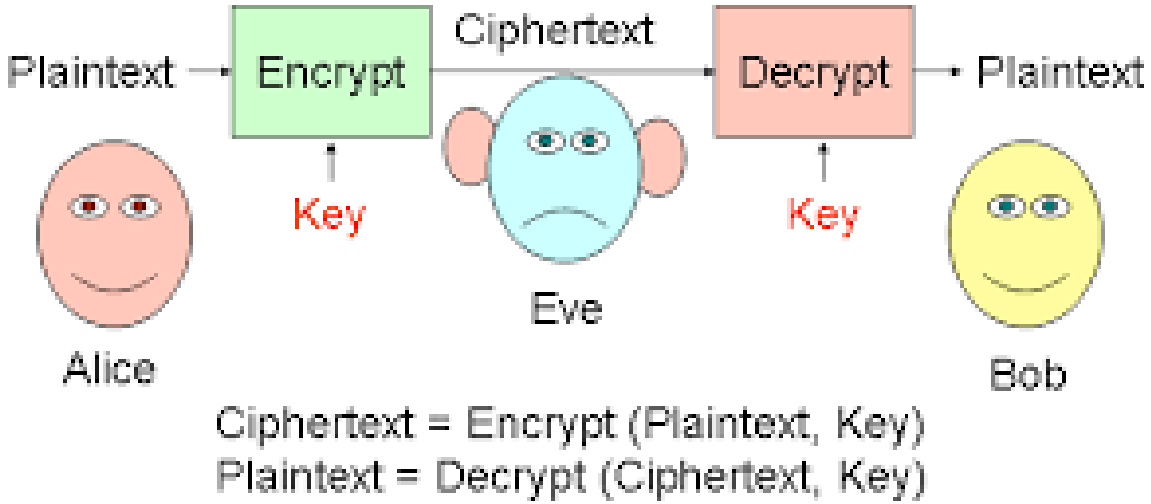

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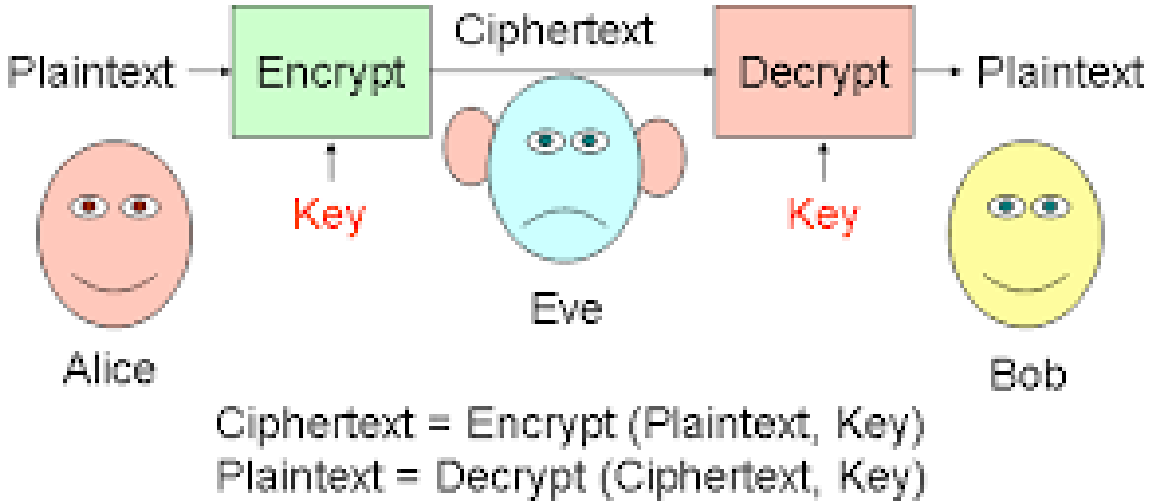

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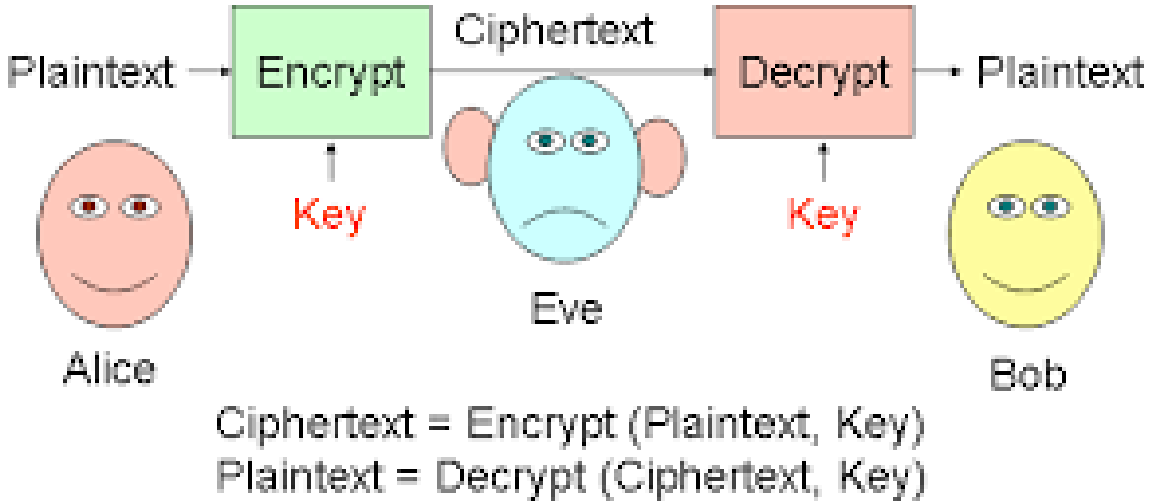

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Bob recovers : Plaintext = Decryption (Message, Bob's private-key)

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Authentication/Non repudiation using digital signatures

Leveraging Hardness for Encrypt/Decrypt function

Factorization

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(52 digits)

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Diffie-Hellman 1976, Rivest-Shamir-Adleman (RSA) 1978

Asymmetry : Verification by Multiplying is easy but factorization doesn't have an efficient algorithm unless ..

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Public Key Cryptography
Diffie-Hellman 1976, Rivest-Shamir-Adleman (RSA) 1978

Asymmetry : Verification by Multiplying is easy but factorization doesn't have an efficient algorithm unless ..
You can build a reliable large quantum computer

Building Faster Computers

Time and work problems

If 30 people can cut the crop of a 10 acre farm in 15 days, how many work days is needed to cut the crop of a 5 acre field using 20 people ?

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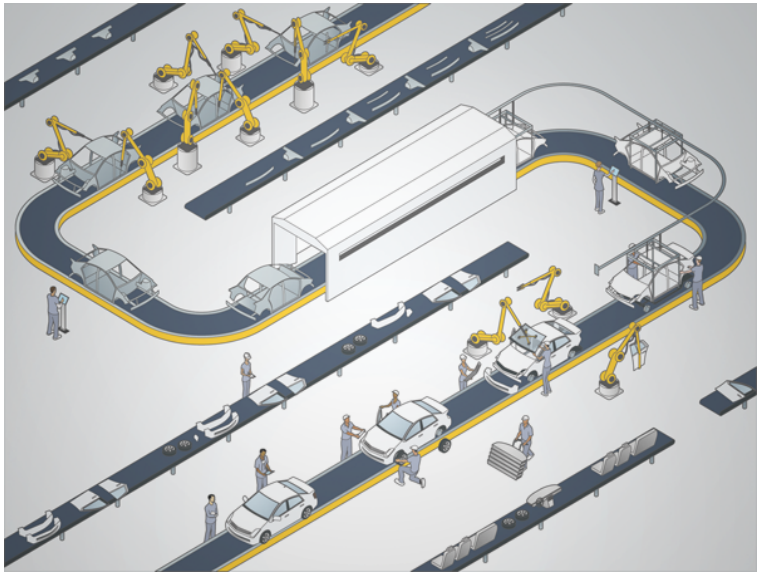
Is this true for all activities ?

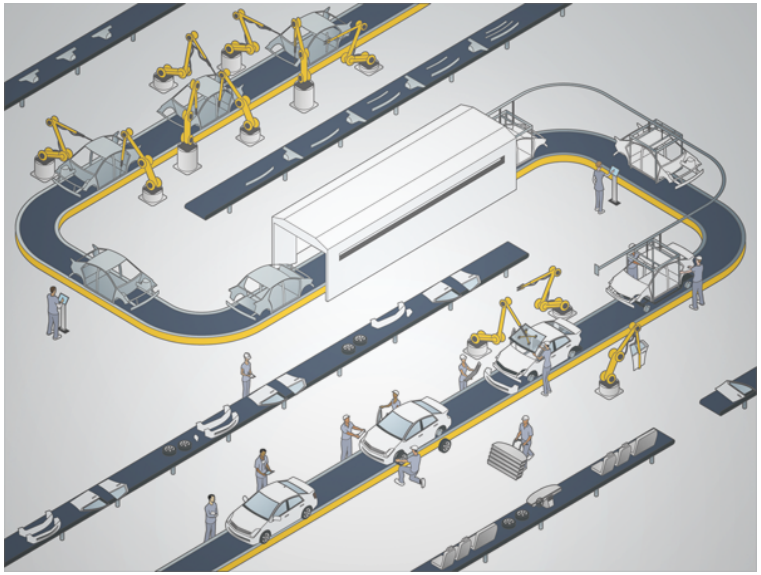
Making Tea

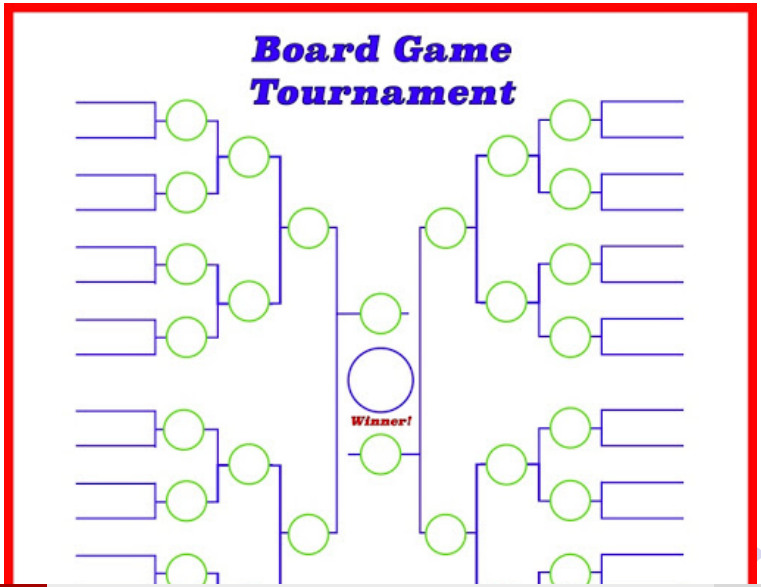
1. Boil water
2. Add Tea leaves
3. Add Milk
4. Add Sugar
5. Pour and drink

Examples of other Activities

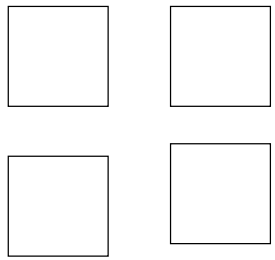


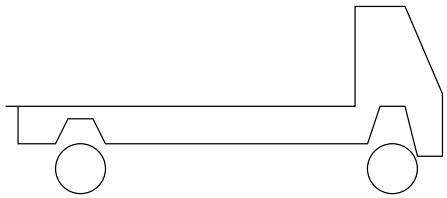
Examples of other Activities



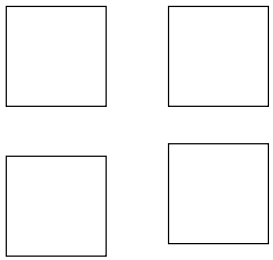


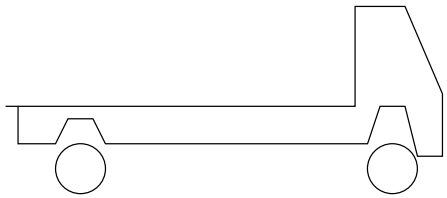
Easy parallelism





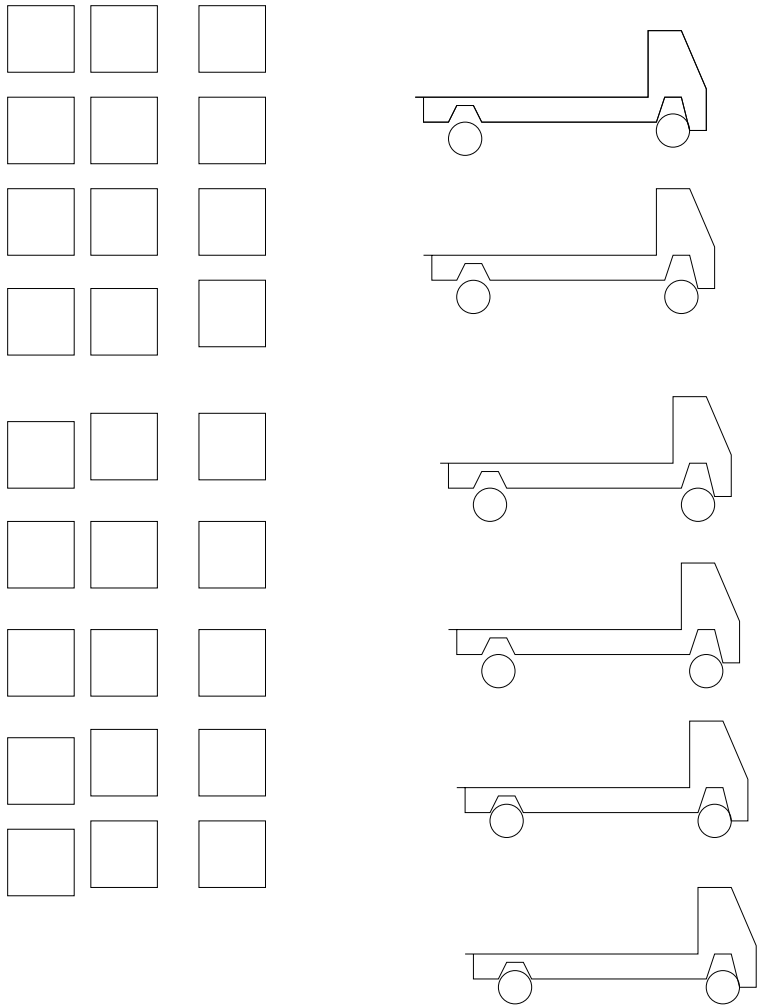
Easy parallelism





More containers, more trips , more time

Easy parallelism



More challenging

ADD

25346437893562435635478598743243593546725363546980508785132036 7

63583733830863024353997265340383616353768935408516253497465093 6

Elementary algorithm : Add and Carry from left to right

More challenging

ADD

25346437893562435635478598743243593546725363546980508785132036 7

63583733830863024353997265340383616353768935408516253497465093 6

Elementary algorithm : Add and Carry from left to right
Number of steps = Number of digits

More challenging

ADD

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63583733830863024353997265340383616353768935408516253497465093 6

Elementary algorithm : Add and Carry from left to right
Number of steps = Number of digits

Multiple processors

CAN RAVAN DO IT FASTER USING MULTIPLE HEADS ??
Multicore/CUDA

More challenging

ADD

25346437893562435635478598743243593546725363546980508785132036763583733830863024353997265340383616353768935408516253497465093635837338308630243539972653403836163537689354085162534974650936

Elementary algorithm : Add and Carry from left to right
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Multiple processors

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One can think about CLOUD as a massive heterogeneous parallel processing environment that is scheduling resources optimally to many jobs running simultaneously.

Where should I do PhD in TCS (Track A FSTTCS)

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Some of the most visible active groups in Algorithms and Complexity

- Algorithms : IIT Delhi , IIT Bombay, IISc , IMSc, ISI Kolkata
- Complexity : IIT Kanpur, IMSc, IISc, TIFR, CMI
- Cryptography, Security : IIT Bombay, IIT Madras, IISc, IIT Kanpur, ISI
- Distributed and Parallel Algorithms: IIIT Hyderabad, IISc
- Quantum Computation : TIFR, IIIT Delhi

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More options

There are smaller active groups and talented individuals in most IITs and other universities - that one can explore by visiting websites as per indvidual interests.